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When the ball strikes the plane, $y = 0$,

$$t = \frac{2v \sin \psi}{g \cos \phi}, \qquad \text{and } x \text{ becomes} \qquad x_1 = \frac{2v^2 \sin \psi}{g \cos^2 \phi} (\cos \psi \cos \phi - \sin \psi \sin \phi),$$

whence,

$$\frac{dx_1}{dt} = v \cos x - \frac{2v \sin \psi \sin \phi}{\cos \phi}, \qquad \frac{dy_1}{dt} = -v \sin \psi.$$

After the rebound,

$$x = x_1 + vt \cos \psi - \frac{2vt \sin \psi \sin \phi}{\cos \phi} - \frac{gt^2 \sin \phi}{2}, \quad y = evt \sin \psi - \frac{gt^2 \cos \phi}{2},$$

t being here reckoned from the instant of rebound. When the ball again strikes the plane, $y = 0$, and

$$t = \frac{2ev \sin \psi}{g \cos \phi} \text{ and } x \text{ becomes } x_2 = x_1 + \frac{2ev^2 \sin \psi \cos \psi}{g \cos \phi} - \frac{4ev^2 \sin^2 \psi \sin \phi}{g \cos^2 \phi} - \frac{2e^2v^2 \sin^2 \psi \sin \phi}{g \cos^2 \phi},$$

or

$$x_2 = \frac{2v^2 \sin \psi}{g \cos^2 \phi} \{(e + 1)\cos \psi \cos \phi - (e^2 + 2e + 1) \sin \psi \sin \phi\}.$$

By the conditions of the problem, $x_2 = 0$, and, hence, $\cot \phi \cot \psi = e + 1$.

II. Solution by the Proposer.

Let v be the velocity of projection; then $v \sin \psi$ and $v \cos \psi$ are the components respectively perpendicular and parallel to the plane; and the components of gravity perpendicular and parallel to the plane are $g \cos \phi$ and $g \sin \phi$. Considering the perpendicular component, the time of flight is twice the time in which the velocity $v \sin \psi$ would be acquired under the action of the force $g \cos \phi$, *i. e.*, $2v \sin \psi/g \cos \phi$. After rebounding, the ball's velocity perpendicular to the plane is $ev \sin \psi$; hence, the time required in returning to the point of projection is $2ev \sin \psi/g \cos \phi$; and the whole time of flight is

$$2 \frac{v \sin \psi}{g \cos \phi} (e + 1).$$

Now, since the motion parallel to the plane is not affected by the impact, the entire time of flight is equal to twice the time in which the velocity $v \cos \psi$ would be produced by the force $g \sin \phi$, *i. e.*, $2 \dfrac{v \cos \psi}{g \sin \phi}$. Therefore, equating the two expressions found for the time, we have

$$2 \frac{v \cos \psi}{g \sin \phi} = 2 \frac{v \sin \psi}{g \cos \phi} (e + 1);$$

whence, $\cot \phi \cot \psi = e + 1$.

NUMBER THEORY.

An excellent solution of 226 by Mrs. Elizabeth Brown Davis should have been reported in the April issue. Editors.

227. Proposed by R. P. BAKER, University of Iowa.

Show that every rational number can be expressed as a finite sum $\sum\limits_{n=m}^{n=m+k} \dfrac{a_n}{n}$, where a_n is either 0 or 1 and m is any positive integer.

Solution by Frank Irwin, University of California.

Since the series $\sum\limits_{n=m}^{n=\infty} \dfrac{1}{n}$ is divergent, we can find a value of r such that our given rational number, N, lies between $1/m + 1/(m + 1) + \cdots + 1/(r - 1)$ and $1/m + 1/(m + 1) + \cdots + 1/r$ (unless indeed N is equal to an expression of this sort, in which case our problem is solved). This reduces the problem to that of expressing $N' = N - [1/m + 1/(m + 1) + \cdots + 1/(r - 1)]$, that is, a (proper) fraction $< 1/r$, as $\sum\limits_{n=r}^{n=m+k} \dfrac{a_n}{n}$.

Let $N' = c/d$. We shall show that we may express c/d as

$$\frac{1}{l}+\frac{c_1}{d_1},$$

when l is $> r$ (else would c/d be $> 1/r$), c_1/d_1 is $< 1/l$, and $c_1 < c$. If d is divisible by c, the thing is done ($c_1 = 0$). Otherwise, let

$$d = (l-1)c + m, \quad 0 < m < c.$$

Then

$$\frac{c}{d} = \frac{1}{l} + \frac{c-m}{dl}$$

is an expression of the kind desired, since $(c-m)/dl < c/dl < 1/l$ (for c/d is a proper fraction).

Now apply the same process to c_1/d_1, which is $< 1/l$:

$$\frac{c_1}{d_1} = \frac{1}{l_1} + \frac{c_2}{d_2}, \qquad l_1 > l.$$

Continuing in this way, we shall eventually get for N' an expression of the kind we are seeking:

$$N' = \frac{c}{d} = \frac{1}{l} + \frac{1}{l_1} + \cdots + \frac{1}{l_s},$$

where

$$r < l < l_1 < \cdots < l_s;$$

for the process must come to a stop since the numerators c_1, c_2, $\cdots$ continually decrease (the process being accelerated whenever one of the fractions c_i/d_i reduces).

The procedure may be described in a few words: to keep adding to our set of fractions the largest one available (*i. e.*, one that has not already been used and whose denominator is not $< m$) that will not make their sum greater than the given rational number.

It is, perhaps, interesting to illustrate the methods employed by a numerical example. Let it be required to express $7/22$ as $\sum_{n=10}^{n=10+k} \frac{a_n}{n}$.

$$\frac{7}{22} = \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{29}{660}, \quad \text{where} \quad \frac{29}{660} < \frac{1}{13};$$

$$\frac{29}{660} = \frac{1}{23} + \frac{7}{15180}; \quad \frac{7}{15180} = \frac{1}{2169} + \frac{3}{32925420} = \frac{1}{2169} + \frac{1}{10975140};$$

the numerators 29, 7, 3 forming a decreasing series, and the last "remainder," 3/32925420, reducing to lower terms.

Our result then is:

$$\frac{7}{22} = \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{23} + \frac{1}{2169} + \frac{1}{10975140}.$$

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

In April, 1915, there was published in this department of the Monthly (Vol. XXII, pp. 119–121) a summary of remarks made by Commissioner Snedden of Massachusetts at the Cincinnati meeting of the N. E. A., Feb. 25, 1915, on the topic "Can algebra and geometry be reorganized so as to justify their retention for high-school pupils not likely to enter technical schools?" These remarks